Some aspects of crack growth and failure in fibre reinforced composites

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A theoretical analysis, previously developed to deal with the machanics of matrix cracking in unidirectional composites and with transverse ply cracking in cross ply laminates, has been developed further to deal with the tensile failure of unidirectional fibrous composites in with the fibres have a known distribution of strengths. It is proposed that, under the application of a tensile load, stable transverse cracks are formed which originate from regions of initial damage and which become unstable at some critical strain value. The model takes account of various parameters including the interfacial fibre/matrix debonding energy, the residual frictional shear strength of the debonded interface and the elastic properties of fibres and matrix. Comparisons are made between the predictions of the model and the observed failing strains of the 0° plies in carbon fibre polymer matrix laminates. The relevance of the model to the study of delayed fracture in fibrous composites is discussed. The modification of this model, previously developed to describe crack growth in the transverse plies of 0°/90° laminates, is used to predict the initial cracking strains for a wide range of CFRP laminate geometries and initial crack sizes. Some aspects of the mechanics of crack extension across interply interfaces are discussed.

1. Introduction

There are many possible modes of failure in fibre reinforced composites and the complexity of the micromechanics of these processes makes it impossible at the present time to evolve a comprehensive and exact theoretical analysis of them. It is possible however to develop analytical models based on simplifying assumptions. In this paper the mechanics of some particular failure processes are analysed by first assuming the form of the strain field extending over a fairly large area around a crack or region of damage and then calculating from the postulated strain field the energy release rate with increasing crack length. Consideration is given to the mechanics of the growth of a crack bridged orthogonally by reinforcing members and loaded in tension in a direction parallel to their alignment. In this way the necessity to describe the form of the strain field close to the tip of the propagating crack is avoided. This is of particular convenience in the case of fibre reinforced systems where the occurrence of cracking parallel to the fibres and the development of damage zones near the tip of the primary crack make it difficult to define the stress field in this region. The model accommodates a limited amount of splitting parallel to the fibres since the strain field around the primary crack is assumed to be divided into a number of parallel independent segments. However, the model is only valid if matrix splitting parallel to the fibres at the crack tip occurs over distance which are short compared with the length of the primary crack. If this is not the case the elastic relaxation of the material will not be confined, as assumed, to a localized region in the general vicinity of the crack. The model has been used previously to analyse the mechanics of the growth of a crack in the following systems: reinforced sheet metal [1], unidirectionally reinforced brittle matrix composites [2], and the cracking of the transverse plies in 0°/90° cross ply laminates [3].

In Section 2 the basic analytical model used is given in outline. In Section 3 comparisons are

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made between the predictions of the theory and the observed inhibition of matrix cracking in steel wire reinforced epoxy resin composites reported by Cooper and Sillwood [4]. The theory is modified in Section 4 to deal with the mechanics of the extension of a region of damage caused by the localized failure of a number of fibres. The predictions of this analysis are compared with the observed tensile strengths of unidirectional systems and the mode of failure in corrosive environments which would be predicted from the model is discussed.

A version of the model has previously been designed to deal with the cracking of a transverse ply in a cross ply laminate and shown to predict results in close numerical agreement with experimental observations [3]. In Section 5 this version is used to predict the cracking behaviour of a wide range of cross ply CFRP (carbon fibre-reinforced plastic) systems containing initial cracks of various sizes and a computational error made in a previous paper is corrected. In Section 6 the general validity and limitations of the various versions of the model are discussed and related to observed fracture processes in fibrous composites materials.

2. The analytical model

The classical Griffith [5] expression defining σ_c the critical stress for unstable growth of a crack of half length, a, in an isotropic elastic plate in plane stress and subjected to a remote tensile load applied in a direction perpendicular to the major axis of the crack is given by,

$$\gamma_p = \frac{\pi a \sigma_c^2}{E} \tag{1}$$

where γ_p is the work of fracture of the material at the crack tip and E is its Young's modulus. Equation 1 is obtained by integrating the strain field around the crack as defined by elasticity theory, differentiating it with respect to increasing crack length and equating this with the work being done in rupturing the material at the crack tip. The most elementary physical model of the stress field around the crack which yields the same numerical result is one in which a zone around the crack equal in area to twice that of a circle having the crack as its diameter has relaxed completely. The material outside this zone is subjected everywhere to a uniform stress σ_c. The zone can be assumed to be elliptical in shape in which case its major axis is twice the length of the crack. The assumption of a sudden transition at the edge of the elliptical boundary from zero stress to the general stress σ_c carried by the material is clearly very far from physical reality. A better physical approximation can be achieved by assuming a uniform increase in stress in the direction of the applied load from zero at the crack face to σ_c at the edge of the partially relaxed zone. If the shape of this zone is again assumed to be elliptical, its major axis has to be increased to three times the crack length in order that the amount of strain energy released from within the elliptical zone should be numerically equivalent to that derived from elasticity theory. If the applied stress generates a strain ϵ_β, the strain gradient for this model has a minimum value of $\epsilon_\beta/3a$ along the major axis of the partially relaxed elliptical zone. This strain gradient increases as the crack tip is approached becoming infinetely large at the crack tip.

The influence of crack bridging reinforcing fibres can be easily calculated for this simple model. The fibres are assumed to cross the crack at right angles to its faces and the external load is applied in the direction of fibre alignment. The fibres carry an enhanced strain where they bridge the matrix crack and the additional load carried by the crack bridging fibres is assumed to be transferred back to the matrix at a constant interfacial shear stress value of τ. Thus the strain carried by the crack bridging fibres decreases linearly with increasing distance from the crack faces. The crack bridging fibres inhibit the elastic relaxation of the matrix on each side of the crack so that the matrix strain is increased over the values which would have been developed in the absence of the reinforcing fibres.

One half of the strain field which would be developed around a matrix crack bridged by fibres on the basis of this argument is shown in Fig. 1 where strain values are indicated on the vertical axis. The diagram shows that the strain carried by fibres and matrix must be equal at some particular distance from the crack face. It is assumed here that no further stress transfer takes place when this condition has been reached so that the strain carried by fibres and matrix remains constant at increasing distances from the crack face. Eventually a position of the strain field, which would have been developed in the absence of the reinforcing fibres, is reached. It is assumed that, beyond this point, the strains carried by fibres

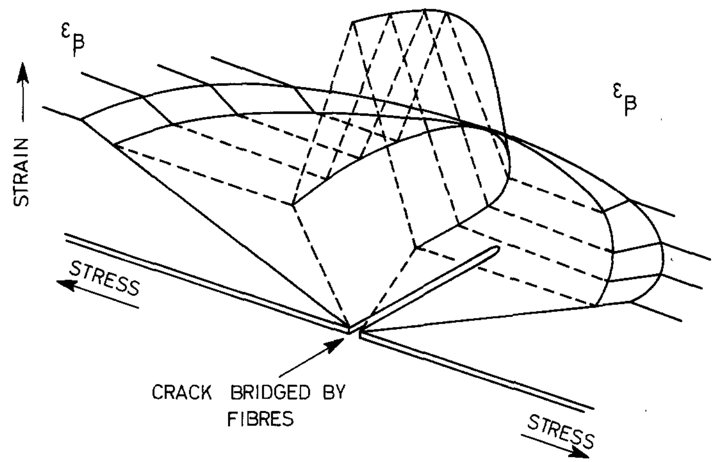

Figure 1 Schematic illustration of one half of the assumed strain field developed around a matrix crack bridged by reinforcing fibres. Strain values plotted on the vertical axis. Full lines indicate the strain field which would be developed in the absence of the reinforcing fibres.

and matrix follow this strain field up to the edge of the elliptical zone. Since all of the perturbations of the strain field are assumed to take place within the elliptical zone it follows that the length of the crack bridging fibres traversing the zone must be unchanged by the occurrence of the matrix crack. This boundary condition enables the geometry of the strain field illustrated in Fig. 1 to be calculated, and an arbitrary section through the strain field parallel to the fibres is shown in Fig. 2. Chemical debonding is assumed to occur over the stress transfer length $0L_1$, adjacent to the crack faces, over which the strains in fibre and matrix are different. Following chemical debonding the differential movements occurring over this debonded interface result in frictional energy losses.

The computation of the rate of release and absorption of strain energy with increasing crack length is carried out in the following way. The elliptical zone around the crack is assumed to be divided into a number of parallel independent zones each aligned with the loading direction. The strain energy released and absorbed by each strip (compared with the initial uncracked matrix condition) is then calculated and this is summed numerically over half of the elliptical zone. This calculation is then repeated for a small increase in crack length to enable the rates of increase and absorption of strain energy with increasing crack length to be obtained. Since the work of fracture and volume fraction of the matrix is also known, the critical crack length can be calculated at which the rate of release of strain energy becomes equal to the rate at which energy is being absorbed by the various processes. Alternatively, for a crack of fixed length, the bulk strain values, ϵ_β, can be obtained at which the crack becomes unstable.

The section through the assumed strain field illustrated in Fig. 2 has been shown [1] to be des-

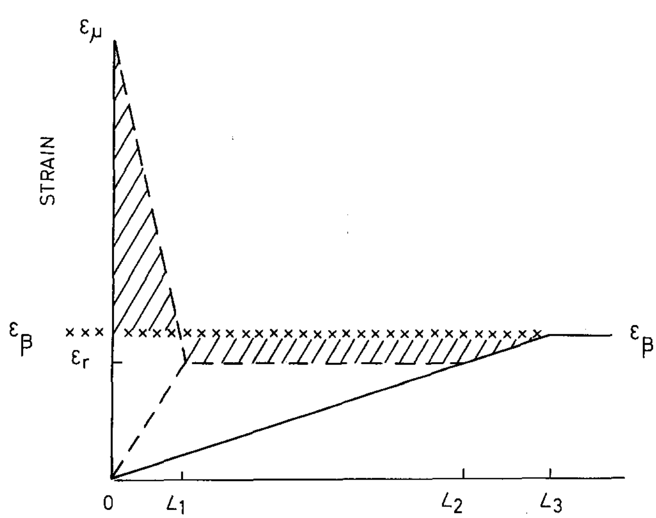

Figure 2 Section through a quadrant of the strain field. Uniform strain in composite with uncracked matrix indicated thus X X X X. Shaded areas indicate the amount of fibre extension and contraction generated by the matrix crack.

cribed by the following equations:

$$\epsilon_r = \{\epsilon_\beta L_3/[Q(P+\epsilon_\beta/L_3)^{-2} + L_3/\epsilon_\beta]\}^{1/2}$$
$$\epsilon_\mu = L_1\,(P+Q+\epsilon_\beta/L_3)$$
$$L_1 = \epsilon_r\,(P+\epsilon_\beta/L_3)^{-1}, L_2 = L_3\,\epsilon_r/\epsilon_\beta$$
$$L_3 = 3\,(a^2-z^2)^{1/2} \quad (2)$$

where $P = 2V_f\tau/E_m V_m r$ and $Q = 2\tau/E_f r$. The values of ϵ_r, ϵ_μ, ϵ_β, L_1, L_2 and L_3 are defined in Fig. 2 and the value of z defines the distance of the section considered from the centre of the crack. V_f and V_m are the fibre and matrix volume fractions, respectively, and the fibre diameter is $2r$.

The strain energy released by a parallel sided strip width δ_z and of unit thickness positioned at a distance z from the centre of the crack is given by δW_{Rz} where,

$$\delta W_{Rz} = [E_c\epsilon_\beta^2 L_3/2 - E_c\epsilon_\beta^2\,(L_3^3 - L_2^3)/6L_3^2 - E_c\epsilon_r^2\,(L_2 - L_1)/2 - E_m\epsilon_r^2 L_1/6 - V_f E_f\,(\epsilon_\mu^2 + \epsilon_\mu\epsilon_r + \epsilon_r^2)L_1/6]\,\delta z. \quad (3)$$

where E_f and E_m are the fibre and matrix volume fractions, respectively, $E_c = E_f V_f + E_m V_m$ and the other symbols used have their usual meanings.

From Equation 3 the strain energy released over two quadrants of the elliptical zone on each side of the crack can be calculated by numerically integrating the strain energy released from a number of parallel strips as previous described. Sufficient accuracy is usually obtained if the zone is assumed to be divided into five parallel strips. The rate of release of strain energy with increasing crack length can then be obtained by numerical differentiation.

Over a distance L_1 from the face of the crack differential movement occurs between the fibres and the surrounding matrix. The energy absorbed by one cylindrical fibre over this distance can be shown to be given by,

$$\pi r\tau\epsilon_\mu L_1^2/3 \quad (4)$$

so that over a parallel sided strip, width δz and unit thickness, within the quadrant of the partially relaxed elliptical zone we have for the absorption of energy by frictional losses,

$$\delta W_{Az}' = V_f\tau\epsilon_\mu L_1^2\delta z/3r. \quad (5)$$

Hence the energy absorbed over two quadrants can be obtained by numerical integration. Again the rate of energy absorption by frictional losses as the crack extends can be obtained by numerical differentiation. Energy is also absorbed in the rupturing of the matrix – the rate of energy absorption per unit increase in crack length for a composite of unit thickness being given by $V_m\gamma_p$ where γ_p is the work of fracture of the matrix.

Work may also be expended in rupturing the fibre/matrix interfacial bond over the distance L_1 prior to the development of frictional losses over this length as displacements occur at the debonded interface. The surface area of each fibre which has to become debonded is $2\pi rL_1$. The number of fibres per unit cross section of the composite is $V_f/\pi r^2$ so that, if the work of fracture for debonding is G_d the work done on each side of the crack per unit increase in crack area will be given by,

$$4V_f L_1\,G_d/r. \quad (6)$$

The critical value of ϵ_β at which a matrix crack of arbitrary length will become unstable is obtained by computing the strain energy release rate for successive small increases in the value of ϵ_β until the rate of release of strain energy with increasing crack length becomes equal to the sum of all of the energy absorbing terms.

3. Suppression of matrix cracking in steel wire reinforced epoxy resin composites

The suppression of matrix cracking in unidirectionally reinforced steel wire/epoxy resin composites was reported by Cooper and Sillwood [4] to occur under certain conditions. The temperature of their experimental samples was reduced progressively to 77 K using a liquid nitrogen bath. This produced a tensile strain of about 0.007 in the resin matrix, as a consequence of differential thermal contraction, and in the case of most samples tested this resulted in the development of an array of parallel cracks in the resin matrix perpendicular to the direction of fibre alignment. However, in the case of samples containing thin wires of diameter 0.10 and 0.12 mm, present in volume fractions greater than about 50%, matrix cracking was not observed. An analysis of the mechanics of transverse cracking of a brittle matrix has been carried out by Aveston *et al.* [6]. This is based on a consideration of the change in energy associated with the transformation from an initially uncracked state to one in which a crack has traversed the full width of the composite and is hence, effectively, infinitely long. The cracking strain of the matrix, ϵ_{muc}, is given from this analysis as

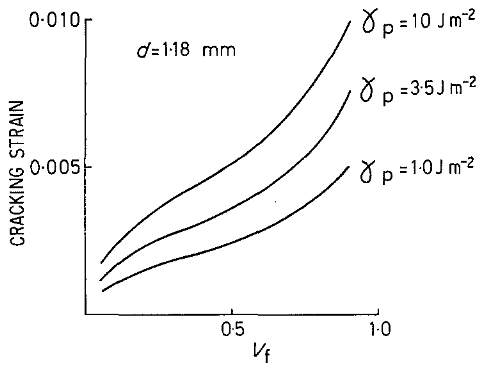

Figure 3 Computed strains at which unstable growth would occur for a matrix crack having a half length of 5 mm. Fibre diameter 1.18 mm.

$$\epsilon_{\text{muc}} = \left(\frac{12\tau E_f V_f^2 \gamma_m}{E_c E_m^2 V_m r}\right)^{1/3} \qquad (7)$$

where γ_m is the surface energy of the matrix, $E_c = E_f V_f + E_m V_f$, $2r$ is the fibre diameter and the other symbols have their usual meanings. Equation 7 predicts high values for ϵ_{muc} as the fibre diameter becomes very small but predicts that ϵ_{muc} will approach zero as V_f approaches zero. Hence, it is regarded as being applicable only when the predicted value of ϵ_{muc} is greater than the intrinsic failing strain of the matrix. Also Equation 7 does not include the crack length as a parameter neither does it deal with any work done in debonding the fibre matrix interface during matrix cracking.

Cooper and Sillwood [4] used values of 1.75 Jm^{-2} for γ_m, the surface energy of the polymeric matrix, but expressed doubts as to the accuracy of this measurement. When this value and the other appropriate constants are inserted into Equation 7 crack suppression is predicted in the matrix for fibre diameters less than 0.18 mm. In view of the uncertainties in the experimental determination of γ_m and τ, Cooper and Sillwood felt the agreement with the predictions of Equation 7 to be quite satisfactory. It was observed that matrix cracks could initiate in resin rich areas of a non-uniform composite containing fine wires but would not enter regions having high fibre volume fractions.

In Figs. 3 and 4 the matrix strain at which a crack having a half length of 5 mm would propagate is computed from the theory outlined in Section 2. This is shown as a function of fibre volume fraction and for different values of γ_p (10, 3.5 and 1.0 Jm^{-2}). Here γ_p is taken as the effective work of matrix fracture and equal to $2\gamma_m$. The other parameters are the same as those used by Cooper

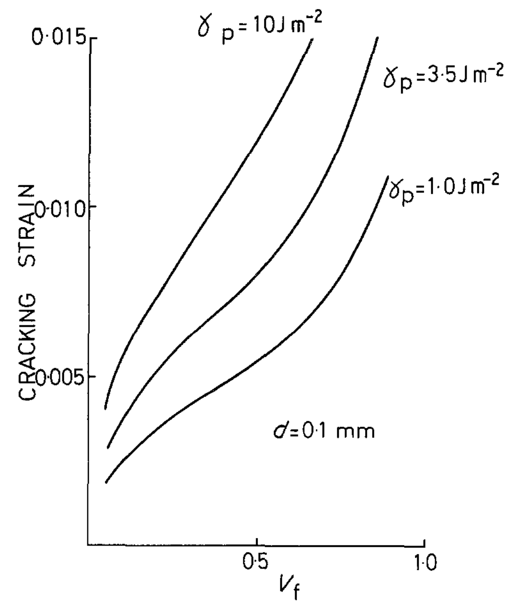

Figure 4 Computed strains at which unstable growth would occur for a matrix crack having a half length of 5 mm. Fibre diameter 0.1 mm.

and Sillwood [4] in their computations. The analysis indicates that, at fibre volume fractions of 50%, matrix crack growth would not be expected in a system containing wires 0.1 mm in diameter at a matrix tensile strain of 0.007 (Fig. 4) unless the value of γ_p becomes very low approaching 1.0 Jm^{-2}. Conversely, in the system containing 50% of wires 1.18 mm in diameter (Fig. 3), matrix crack extension would be expected at this strain value even for values of γ_p of 10 Jm^{-2}. The chemical debonding energy of the fibre matrix interface is assumed here to be zero in order to conform with the calculations carried out by Cooper and Sillwood. The computed matrix cracking strain would be enhanced by the inclusion of this parameter.

According to the analysis presented here the strain at which a matrix crack will propagate will be a function of the crack length. However, at fibre volume fractions of 50% the calculations indicate that, for this particular system, the strain for unstable crack extension will be sensibly independent of crack length except for very short cracks having a length comparable with the fibre diameter. The model is clearly invalid for these conditions. However, as the fibre volume fraction is diminished, the strain for crack extension becomes more sensitive to crack length becoming inversely proportional to the square root of the crack length as the fibre volume fraction tends to zero.

4. Failure of the 0° ply in a 0°/90° laminate

Unless the relative thickness of the longitudinal plies in a 0°/90° laminate is very small, cracking of the transverse plies does not immediately cause the laminate to fail. Generally the load applied to the composite is supported by the longitudinal plies until they in turn fracture at their failing strain. In the case of the cross ply composites studied by Bailey *et al.* [7], this occurred at a fairly consistent strain value. The same authors observed the onset of acoustic output in the 0° CFRP material at strains of about 60% of the failing strain. The acoustic output increases rapidly as the composite failing strains are approached and is generally assumed to be caused by fibre fracture.

The model previously proposed to account for the mechanics of matrix fracture in fibre composites, outlined in Section 2, is modified here to deal with the propagation of a crack from a localized region of fibre fracture. Adventitious mechanical surface damage to a composite would be expected to cause localized fibre fracture extending to some little distance below the composite surface. Also regions in which fibres have fractured, essentially within a plane, either before or during fabrication or during initial loading can be expected to occur inside the body of a composite structure.

It is assumed here that localized regions of damage are caused by fibre failure and that these develop into stable cracks bridged by intact fibres. The lengths of the stable cracks increase as the strain on the composite is increased until unstable extension occurs by the fracture of the still intact crack bridging fibres. The possibility of composite failure due to the linking of stable transverse cracks through failure of the matrix in longitudinal shear is not considered. Other forms of failure such as longitudinal splitting (shear back) and the development of damage zones which are usually associated with the presence of a notch of appreciable size and/or cyclic loading are also not considered in this analysis.

4.1. Theory

The idealized physical model, upon which the analysis developed here is based, is illustrated in Fig. 5. A region of damge of arbitrary size is considered to be present and this is characterized by the local fracture of an arbitrary number of fibres. The ends of the broken fibres are assumed to be located within a single plane so that the region of damage can be assumed to behave as a crack bridged by those fibres which remain intact. The broken fibres and the polymeric binder are assumed to form the "matrix" and the properties of the broken fibres and the polymeric binder are summed to give the average work of fracture and elastic modulus of the matrix measured in the direction of fibre alignment, (which is also the loading direction). Hence the elastic modulus of the "matrix" increases as the proportion of fractured fibres increases.

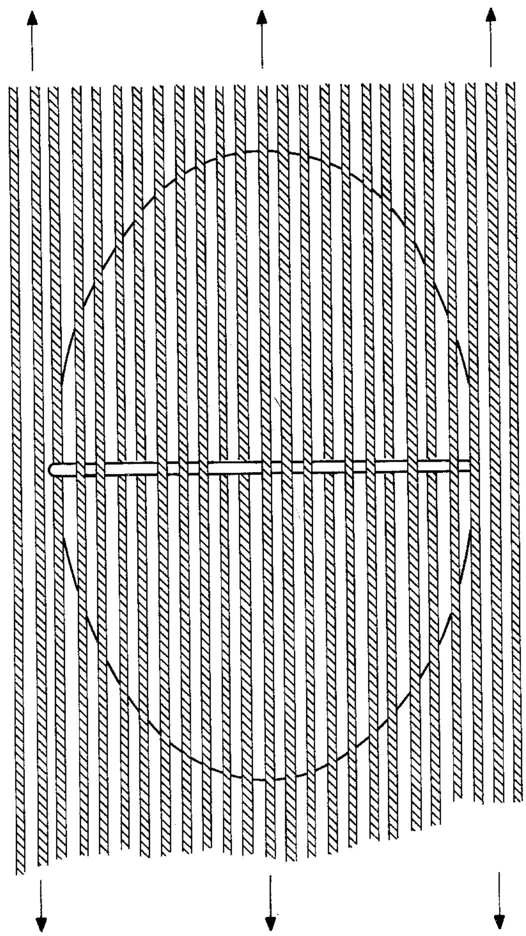

Figure 5 Schematic illustration of a planar zone of damage surrounded by partially relaxed material.

The crack may initiate at a relatively low stress value in a small region in which a high proportion of the fibres have fractured. As the crack extends it is assumed to encounter and bypass unfractured fibres so that the proportion of crack bridging fibres increases. During the growth of the matrix crack work has to be done in the further debonding of the interface between the crack bridging fibres and the matrix and in rupturing the matrix and those fibres which fracture. Also work is expended in frictional losses occurring as a consequence of displacements between the crack bridging fibres and the surrounding matrix. These processes stabilize the crack at a given composite strain value. The crack bridging fibres carry an enhanced strain so that eventually the crack will become unstable by the sequential failure of crack bridging fibres. This critical composite strain value is governed by the fibre strength distribution and the various physical processes associated with the mechanics of failure.

In the system considered here work is expended in rupturing the fibre/matrix interfacial bond over the distance $0L_1$ (Fig. 2), as defined in Equation 6, prior to the development of frictional losses. This term relates only to the fibres which remain intact and bridge the crack and has to be added to the matrix work of fracture term in deriving an energy balance as the condition for crack growth. We first consider the stable growth of a matrix crack as ϵ_β is increased.

It is necessary to include the work of fracture of that proportion of the fibres which fracture as the crack extends. We assume initially that the proportion of fibres which have fractured and which do not, therefore, bridge the crack will remain the same as the crack extends. If N is the proportion of fibres which have fractured and V_{ft} is the total volume fraction of fibres in the composite then the proportion intact will be given by $V_{ft}(1-N) = V_f$ since the fractured fibres are considered to form part of the matrix. Hence, the matrix elastic modulus E_m is now given by,

$$E_m = (V_p E_p + N V_{ft} E_f)/V_m \qquad (8)$$

where $V_m = (1 - V_f)$. V_p is the volume fraction of the polymeric material so that $V_p = (1 - V_{ft})$. E_p is the elastic modulus of the polymeric material.

Energy losses developed by fibres which fracture before the point of crack instability is reached (with N held constant) are not considered to contribute to the rate of energy loss at instability). However, the work of fracture of those fibres which have to fracture in allowing the crack to extend by an infinitesimal amount at instability with N held constant is taken into account. Thus the matrix work of fracture is given by,

$$G_m = (V_p W_{pf} + N V_{ft} W_{ff})/V_m \qquad (9)$$

where W_{pf} is the work of fracture of the polymeric material and W_{ff} is the work of fracture and the fibres themselves.

The peak strain carried by the crack bridging fibres is denoted by ϵ_μ (see Fig. 2 and Equation 2). Its value is relatively insensitive to the distance of any particular fibre from the centre of the crack except for positions very close to the crack tip. In order to simplify the calculation, the strain carried by a fibre at a point located mid-way between the centre of the crack and the crack tip is regarded as representative of the strains carried by all of the crack bridging fibres. Given a range of fibre failing strains the proportion of fibres which would have been expected to fracture over the length of the crack as a consequence of the enhanced strains carried by them can be calculated. This is defined as FN. Calculations have been made [8] of the mechanics of transverse crack growth and tensile failure of a composite containing a 60% volume fraction of unidirectionally aligned type III carbon fibres. The fibre elastic modulus was taken as 200 GPa and the fibre failing strains were assumed to be uniformly distributed between 0.01375 and 0.01875. This range of failing strains is similar to that observed by Barry [9]. The proportion of fibres fractured, FN, at a representative strain values of ϵ_μ is given by

$$FN = (\epsilon_\mu - 0.01375)/0.005 \quad \text{for } 0 < FN < 1.0. \qquad (10)$$

The half crack length, a, was set at three arbitrary sizes, 0.001, 0.0002 and 0.00005 m. Four values of the work of interfacial debonding G_d were taken: 0, 5, 15 and 100 J m^{-2}. Two values of residual frictional interaction between the fibres and the matrix after debonding were used. These were 1 and 10 MPa. The elastic modulus of the polymer (E_p) was taken as 4 GPa, its work of fracture (W_{pf}) as 200 J m^{-2} and the work of fracture of the fibres (W_{ff}) as 150 J m^{-2}.

For each combination of the above properties the critical strain for crack propagation was first computed for a value of N of unity. This condition, therefore, corresponded to one in which all the fibres had fractured over the length on the arbitrary crack. The critical strain for crack extension was obtained by incrementally increasing the value of ϵ_β until the computed rate of release of strain energy with increasing crack length became equal to the corresponding rate of absorption of energy. This latter term was a sum of the energy being absorbed in the further debonding of the interfaces of the intact fibres, the further frictional losses incurred by displacements at the interfaces after debonding, the work of fracture of the polymer matrix and the work of fracture of the fracturing fibres.

When N is unity the critical strain values for crack extension is small and in computing its value the assumption is made that the value of N will remain constant as the crack extends. In general this will not be the case and the crack will become bridged by an increasing proportion of intact fibres as it extends. The effect of the increased proportion of crack bridging fibres is to reduce the critical strain for further crack propagation below that required for crack initiation so that crack growth ceases. Stable crack extension then occurs if the composite strain ϵ_β is increased. Numerical values can be obtained for these conditions using the analysis outlined above quite straightforwardly if it is assumed that the fractured fibres are uniformly distributed over the length of the crack. The situation is illustrated in Fig. 6. For example, a crack having an initial half length of 0.05 mm over which all the fibres have fractured will propagate at a composite strain of about 0.003. A crack four times this initial length (half length 0.2 mm) will be stable at this strain value if an many as 70% of the fibres encountered by the crack have fractured, (for $\tau = 10$ MPa and $G_d = 0\,\text{Jm}^{-2}$). If the initial crack encounters only intact fibres as it extends the proportion of fractured fibres averaged over its new length of 0.2 mm would be only 25%. Clearly for this particular situation the initial crack, propagating at a constant strain of 0.003, will become stable before reaching a half length of 0.2 mm. An increase in composite strain over the value 0.003 will then cause the crack to extend again to a new stable length corresponding to the enhanced strain value. Stability will have been obtained because, although the crack has increased in length, the proportion of fractured fibres over its length will have been further reduced. Thus stable crack growth at increasing composite strain values will be expected provided a sufficiently high proportion of fibres remain intact and bridge the crack faces.

Eventually, at some particular crack length and composite strain value, the theoretical model predicts that the additional strain carried by the crack bridging fibres will cause the fracture of a sufficiently high proportion of them to allow instability and uncontrolled crack extension by the progressive fracture of the remaining crack bridging fibres. The condition for this form of instability has been calculated in the following way. The proportion of broken fibres, N, is first assumed and the effective strain carried by the bridging fibres at the composite strain for stable flaw growth is computed as described above. From this data the proportion of the crack bridging fibres which would have been expected to fracture, FN, is calculated from the known distribution of fibre strengths using equation 10. If $FN < N$, the value of N is reduced and the calculation repeated. This results in an enhanced strain for stable crack extension. (Note that for each incremental reduction in N, the new condition for crack growth is calculated on the assumption that N will remain constant as the crack extends.) The new enhanced value of ϵ_β for crack growth causes an increase in ϵ_μ and hence an enhanced

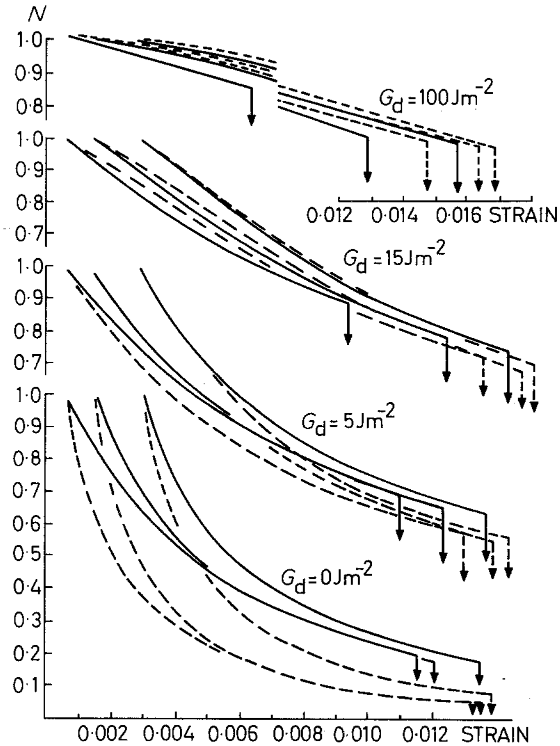

Figure 6 Composite strain values for stable flaw extension. Catastrophic failure due to sequential failure of crack bridging fibres indicated thus ↓. Values of $\tau = 10$ MPa indicated by full lines, τ values of 1 MPa indicated by dashed lines. Computations relate to three flaw sizes ($a = 1$, 2 and 0.05 mm). (Redrawn from [8].)

value of FN. The value of N is again compared with that of FN and the cycle of computation repeated until the two values are equal. For this condition the crack is unstable because any increase in ϵ_β increases the proportion of crack bridging fibres which have fractured thus destabilizing the crack.

4.2. Discussion

The acoustic emission noted by Bailey *et al.* [7], from unidirectional carbon fibre reinforced composites at strains in excess of 0.007, may have been caused by fibre and matrix fracture occurring during the stable extension of small regions of initial damage according to the model proposed here. Final fracture may occur either by the unstable growth of an initially stable crack by sequential failure of the crack bridging fibres or by the linking together of stable cracks on different planes by shear failure of the matrix.

In Fig. 6 the calculated strains for stable crack growth (assuming a constant value of N during crack extension) are shown plotted against the composite strain ϵ_β. The fibre volume fraction is taken as 60% and the fibre diameter 10 μm. The full lines correspond to values of τ, (the residual interfacial frictional shear strength), of 10 MPa and the dashed lines to τ values of 1 MPa. The effects of fibre/matrix debonding energies, G_d, ranging from zero to 100 Jm^{-2} are considered. This composite system corresponds closely to that studied by Bailey *et al.* [7]. As N falls the proportion of crack bridging fibres increases and the strain value for crack extension, for a particular constant value of N, increases. As ϵ_β is increased the strain carried by the crack bridging fibres is increased. However, for the conditions examined, this enhanced strain is insufficient to fracture any of the assumed intact crack bridging fibres until the value of ϵ_β approaches closely the strain for unstable crack growth by sequential failure of the crack bridging fibres. As this critical strain is approached the computed value of FN increases rapidly to approach N. The condition that $FN = N$ defines the critical strain for unstable crack growth by this mechanism and also defines the proportion of fibres which are required to stabilize a flaw of artibrary size. It is apparent from Fig. 6 that this proportion is a function of the debonding energy of the fibre matrix interface. As would be expected, the model predicts an increasing degree of flaw sensitivity, or brittleness, as the coupling between the fibres and matrix is enhanced. The flaw sensitivity increases if both the chemical debonding energy and the residual frictional interfacial reaction are increased. However, it is interesting to note that, for flaws of arbitrary size, enhanced failing strains are predicted by the model if G_d is large and τ is small. The physical reason for this is that for the condition only a few high strength fibres are required to stabilize a flaw.

The smallest half crack size (a) considered in Fig. 6 is 0.00005 m and this corresponds to the failure in one plane of about eight adjacent fibres. If the flaw is located at the surface this corresponds to damage extending to four fibre diameters below the surface. Flaws of at least these dimensions seem very likely to be present. The composite failure strain is not reduced significantly by flaws having a half length up to 0.001 m, providing both G_d and τ are small, but the composite strength is predicted to fall considerably when large flaws are present and when G_d and τ are large.

This effect is predicted to be more apparent in the case of high elastic modulus carbon fibres [8]. The numerical values of the failing strain of the composite system described in Fig. 6 are approximately 0.012 over a fairly wide range of flaw sizes and composite properties. These values are similar to the experimental failing strain values of 0.116 ± 0.004 observed by Bailey *et al.* [7] for a similar unidirectional system. The fracture strains of the 0°/90° laminates investigated by these authors were governed by the failing strains of the 0° plies. These values were again very similar being consistently at a level of about 0.011.

5. Failure of the transverse plies in 0°/90° laminates

A modified version of the strain field theory described above has been developed previously [3] to describe the mechanics of cracking of a transverse ply in a 0°/90° laminate. A through crack of arbitrary length is considered to be present in the transverse ply as indicated in Fig. 7 and the strain at which this crack will propagate is computed. Stress transfer now takes place across the interply interface and Equation 2 can be modified to deal with a laminate by making the following substituations: T_l for V_f, T_t for V_m, E_l for E_f, E_t for E_m and $1/T_lT_{tot}$ for $2/r$, where T_{tot} is the total thickness of the two ply laminate, T_l and T_t are the effective volume fractions of the longitudinal

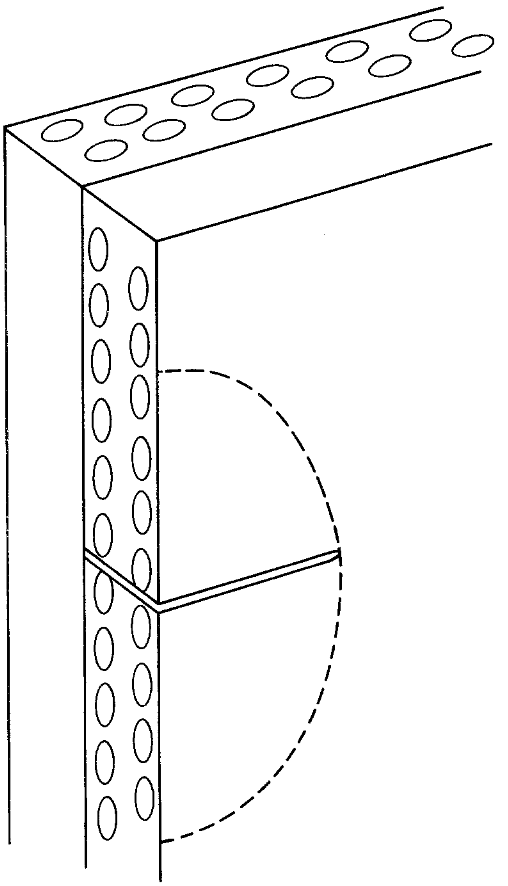

Figure 7 Schematic representation of a "unit cell" two ply 0°/90° laminate showing an edge crack in the 90° ply.

and transverse plies, respectively, and E_l and T_t are their elastic moduli measured in the direction of loading. The value of τ now refers to the effective rate of shear stress transfer across the interply interface. Thus the conversion of Equations 2 to deal with laminates effectively describe a "fibre equivalent system" in which the fibre diameter is given by $4(T_l T_{tot})$ and is thus generally much greater than the diameters of the individual fibres in the plies. The two ply "unit cell" model shown in Fig. 7 can be applied to more complex laminates by setting up an array of "unit cells" each containing one interply interface. If the ply thicknesses are not constant throughout the laminate then the "unit cells" have different dimensions and the theory predicts that, for a fixed initial flaw size, transverse ply cracking will occur at different strain values in the various transverse plies.

It should be noted that the theory assumes a uniform strain throughout the thickness of each ply. Clearly this approximation will not be valid near the interply interface since shear deformation must be generated over appreciable distances on each side of the interface. The mechanism of stress transfer across the interply interface is assumed to operate at a constant value which is a further approximation. Despite these assumptions the simple theory has been shown to be capable of predicting transverse ply cracking strains observed in 0°/90° CFRP laminates [3] over a wide range of laminate geometries including the condition when the 0° ply is reduced to zero thickness. The assumptions made in carrying out these calculations are that cracks having a half length of about 0.2 mm are present in the transverse plies and that the effective constant rate of shear stress transfer across the interply is 10 MNm^{-2}. These assumptions have been adhered to in computing the transverse ply cracking strain over a wide range of two ply 0°/90° "unit cell" laminate geometries. The results of these calculations are shown in Fig. 8 where the 0° ply thickness is varied between zero and 1 mm and the 90° ply thickness between 0.05 and 1 mm. The particular laminate geometries for which experimental data is available [7], reduced to two ply equivalents, are indicated in the diagram. The curves refer to two ply "unit cells" in which the thickness of the 90° ply (TPT) is held constant at the values shown as the thickness of the 0° ply (LPT) is varied. The transverse ply is assumed to contain a crack of half length 0.2 mm and the lower horizontal axis indicates the composite strain at which the crack will propagate. No correction is made for thermal stresses produced by differential contraction of the plies from the curing temperature of the polymeric matrix. The curves converge at the failing strain of the transverse ply (containing a crack of half length 0.2 mm) as the thickness of the 0° ply tends to zero. In Figs. 9 and 10 the same calculations are repeated for crack half lengths of 1 and 5 mm, respectively.

It is of interest to note that for all of the crack sizes investigated maximum stabilization is achieved when the 0° ply thickness (LPT) reaches a value of about 0.2 mm. The calculations indicate that there is only a little increase in the transverse ply cracking strain as the 0° ply thickness is increased above this value. Also when the thicknesses of both the 0° and 90° ply are small (about 0.1 mm) the laminate strain for crack extension becomes insensitive to the length of the crack in the transverse ply.

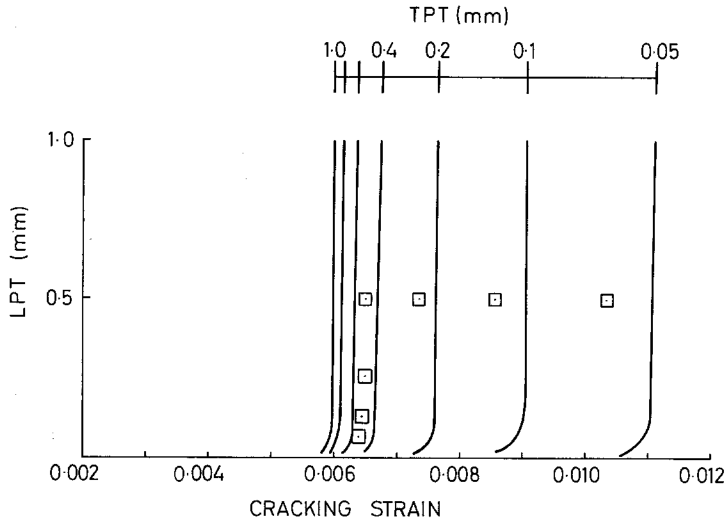

Figure 8 Calculated cracking strains for the 90° ply in a 0°/90° two ply laminate. Crack of half length 0.2 mm aligned parallel to the fibres assumed to be present in the 90° ply. TPT = 90° ply thickness. LPT = 0° ply thickness. Experimental samples studied in [7] shown thus ⊡.

During the course of these computations an error was discovered in previously published calculations of cracking strain values for 90°/0°/90° composites in which a crack is assumed to propagate in only one of the transverse plies [3]. A recalculation indicates that very little difference would be expected in the cracking strain values for crack growth in one or simultaneously in both transverse plies. The two calculated curves together with the experimental values obtained by Bailey *et al.* [7] are shown in Fig. 11.

6. Conclusions

As shown in Section 2 the model predicts with reasonable accuracy the suppression of crack growth in epoxy resin reinforced with steel wires. The observed growth of cracks in resin rich areas, which subsequently become stabilized as they encounter crack bridging fibres, is also predicted by the model. Good correlation between the predictions of the model, modified so as to apply to laminates, and the observed cracking strain of the transverse ply in 0°/90° CFRP laminates is also found. The model can be used to predict the effect of very low proportions of 0° plies and also the behaviour of cracks of various lengths in the transverse plies, and these are described in Section 4. In deriving this version of the strain field model the further assumption that the strains developed are uniform throughout the thickness of each ply is made. On the basis of this assumption the calculated enhanced strains carried by the fibres of the crack bridging 0° ply are insufficient to cause fibre failure over the range of laminate geometries investigated in Section 4 and this result is in agreement with the general observation that cracks in

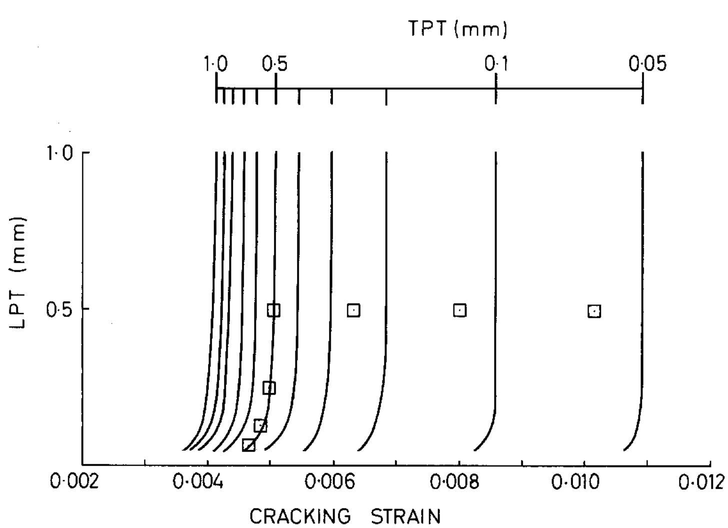

Figure 9 Calculated cracking strains for the 90° ply in a 0°/90° two ply laminate. Crack of half length 1 mm aligned parallel to the fibres assumed to be present in the 90° ply. TPT = 90° ply thickness. LPT = 0° ply thickness. Experimental samples studied in [7] shown thus ⊡.

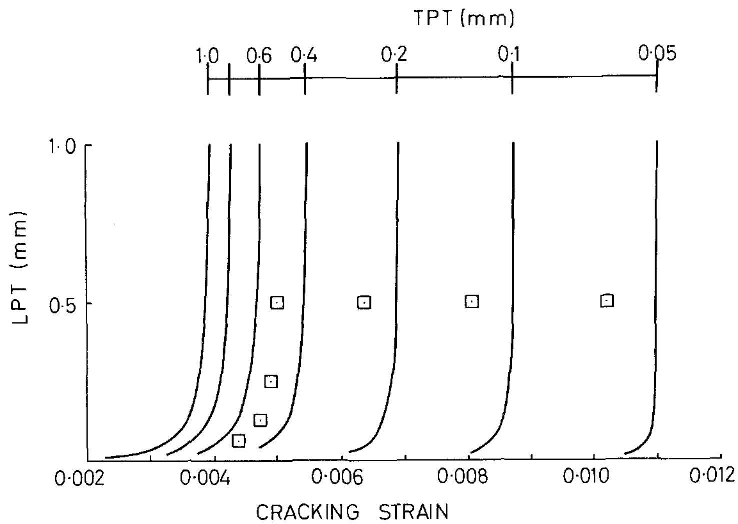

Figure 10 Calculated cracking strains for the 90° ply in a 0°/ 90° two ply laminate. Crack of half length 5 mm aligned parallel to the fibres assumed to be present in the 90° ply. TPT = 90° ply thickness. LPT = 0° ply thickness. Experimental samples studied in [7] shown thus ⊡.

the transverse plies do not enter the longitudinal plies. However, it is to be expected that stresses higher than those computed would be developed in the fibres near to the surface of the 0° plies, because of local perturbations of the assumed strain field, so that localized fibre fracture might, in some cases, occur. In this context it is interesting to note that Reifsnider [10] has reported the localized failure of fibres in 0° plies along lines corresponding to cracks in adjacent off-axis plies in laminates subjected to fatigue loading.

The strain field model is modified in Section 3 to describe the mechanics of transverse crack growth from a localized region of fractured fibres in a unidirectional composite system subjected to a tensile load applied in the direction of the reinforceing fibres. Failure is assumed to proceed via the development of stable cracks which eventually become unstable due to the sequential failure of the crack bridging fibres. This process implies the development of a planar crack and the model predicts the increasing probability of this process as the degree of coupling between fibres and matrix increases in agreement with experimental observations. At reduced fibre matrix coupling levels failure would be expected to occur by the linking together of an array of stable cracks as a consequence of longitudinal shear failure in the matrix. On substituting typical fibre properties into the calculations the model predicts tensile failing strains very similar to those observed for the 0° plies of unidirectional CFRP laminates. The model can be used to predict the effect of fibre strength distributions within a single population of fibres and also, in principle, the effect of different fibre populations (hybrids) on the tensile failure processes occurring in unidirectional fibrous composites. A particular feature of the model is its ability to describe the stabilization of localized regions of damage, extending over dimensions of the order of many fibre diameters which must be present at least on the surface of practical fibrous composites.

It is interesting to note that the strain enhancement, predicted to occur in the crack bridging fibres, would lead to their premature failure when

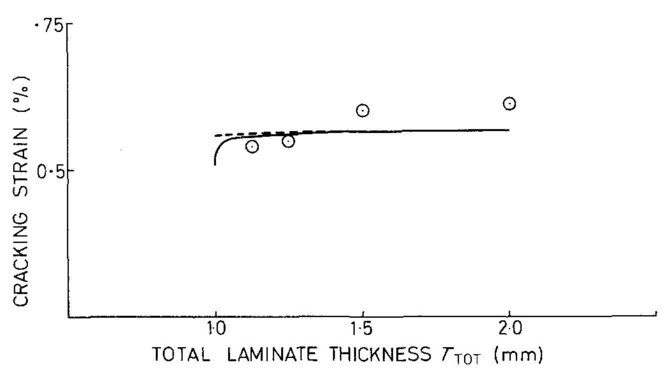

Figure 11 Recalculated transverse ply cracking strains for 90°/0°/90° CFRP laminate. Simultaneous crack extension in both transverse plies indicated thus ——. Crack growth occurring in only one of the transverse plies indicated thus – – –. Initial crack of half length 0.2 mm assumed for both conditions. Experimental values from [7] indicated thus ⊙.

the fibres are subjected to a corrosive environment. Thus the widely observed delayed fracture effects in fibrous composites subjected to aggressive environments are consistent with the predictions of the model.

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